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What will drive global economic growth in the digital age?

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Abstract: This paper provides a theoretical investigation of possible sources of long-run economic growth in the future. Historically, in the industrial era and during the ongoing digital revolution (which began approximately in the 1980s) the main engine of global economic growth has been research and development (R&D), translating into systematic labor-augmenting technological progress and trend growth in labor productivity. If in the future all essential production or R&D tasks will eventually be subject to automation, though, the engine of growth will be shifted to the accumulation of programmable hardware (capital), and R&D will lose its prominence. Economic growth will then accelerate, no longer constrained by the scarce human input. By contrast, if some essential production and R&D tasks will never be fully automatable, then R&D may forever remain the main growth engine, and the human input may forever remain the scarce, limiting factor of global growth. Additional studied mechanisms include the accumulation of R&D capital and hardware-augmenting technical change.

Keywords: asymptotic dynamics; automation; factor accumulation; long-run economic growth; technical change.

1 Introduction

Expansion in the digital sphere is now an order of magnitude faster than growth in the global capital stock and gross domestic product (GDP): data volume, processing power and bandwidth double every 2–3 years, whereas global GDP doubles every 20–30 years. Since the 1980s "general-purpose computing capacity grew at an annual rate of 58%. The world's capacity for bidirectional telecommunication grew at 28% per year, closely followed by the increase in globally stored information (23%)" (Hilbert and López 2011). The costs of a standard computation have been declining by 53% per year on average since 1940 (Nordhaus 2021). The processing, storage, and communication of information has decoupled from the cognitive capacities of the human brain: "less than one percent of information was in digital format in the mid-1980s, growing to more than 99% today" (Gillings, Hilbert, and Kemp 2016). Preliminary evidence also suggests that since the 1980s the efficiency of computer algorithms has been improving at a pace that is of the same order of magnitude as accumulation of digital hardware (Grace 2013; Hernandez and Brown 2020). Corroborating this finding, in the recent decade we have witnessed a surge in artificial intelligence (AI) breakthroughs based on the methodology of *deep neural networks* (Tegmark 2017), from autonomous vehicles and simultaneous language interpretation to self-taught superhuman performance at chess and Go (Silver et al. 2018) as well as highly accurate prediction of protein structures (Jumper et al. 2021).

However, the jury is still out on how (if at all) these tendencies will affect global long-run economic growth in the coming decades. Some economists such as Jones (2002), Gordon (2016) have documented that the trend growth rate of labor productivity and total factor productivity (TFP) has been in fact slowing down since the

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1980s and formulated a hypothesis that the global economy is therefore heading towards secular stagnation. Other economists, such as Brynjolfsson and McAfee (2014); Brynjolfsson, Rock, and Syverson (2019), put forward an alternative hypothesis that the recent slowdown in productivity growth is only temporary and represents a transition phase between the industrial and the digital era, which – not quite coincidentally – just took off in the 1980s. Once the transition phase is over, they say, thanks to the rapidly increasing capacity of digital technologies the trend growth rate in productivity will rebound and perhaps even surpass the one observed in the 1950s–1980s. Finally, futurists and AI researchers, as well as economists who studied the patterns of economic growth over millennia, have pointed out the likelihood of an upcoming technological singularity (Davidson 2021; Hanson and Yudkowsky 2013; Kurzweil 2005; Roodman 2020; Sandberg 2013) – which would imply even greater growth acceleration.

This discussion can be structured using the *hardware–software framework* proposed by Growiec (2019) and further discussed in Growiec (2022a). A key novelty of that approach is to replace capital and labor as key macroeconomic factors of production with two alternative aggregates: hardware ("brawn") and software ("brains"), orthogonal to the traditional distinction (Figure 1). Thanks to this modification, the framework allows for reliable reduced-form modelling of production processes in the digital era, in which an increasing share of cognitive tasks (information processing, communication, prediction, decision making, etc.) is carried out outside of the human brain.

In the current paper I construct a reduced form growth model with hardware and software as inputs, extending the original Growiec (2019) framework to accommodate a range of alternative assumptions regarding automatability of production and research and development (R&D) tasks as well as the structure of the R&D process. With this in hand, I study the sources and dynamics of global economic growth in the long-run future across a range of feasible scenarios of how the digital era may unfold. Each studied scenario places our global future somewhere along the axis: secular stagnation – continued growth at the current pace – growth acceleration – technological singularity.

Specifically I consider the following research questions.

- **Full versus Partial Automation**. *How is the long-run growth rate and its driving force affected whether or not all essential tasks can be automated?*
- **Automation of Production versus R&D**. *How is the long-run growth rate and its driving force affected if only production, but not R&D tasks can be automated? And conversely, what if only R&D, but not production tasks can be automated?*

I also ask two additional questions, aimed at assessing the robustness of my results to two important modifications of the considered model.

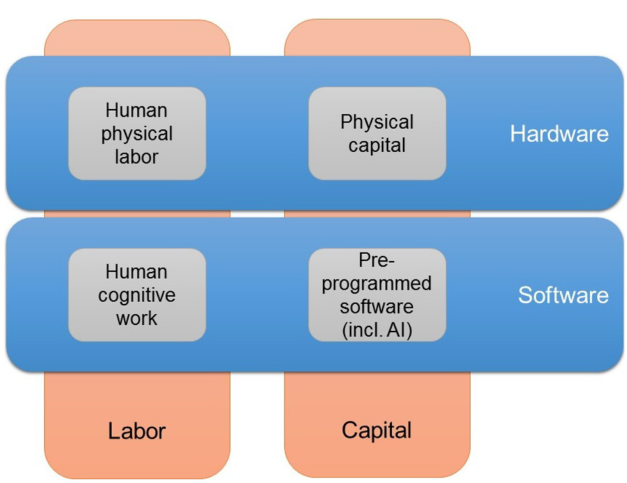

Figure 1: Factors of production in the hardware–software framework.

- **R&D Capital**. *How is the long-run growth rate and its driving force affected whether or not machines (physical capital) can be used in the R&D process?*
- **Hardware-Augmenting Technical Change**. *How is the long-run growth rate and its driving force affected whether or not technical change can be (at least partly) hardware-augmenting?*

Among all considered scenarios, two are probably most likely to happen in reality. First, if full automation eventually turns out to be impossible – that is, if human cognitive work will always be necessary in certain essential tasks – then over the coming decades the pace of global economic growth will likely remain in the ballpark of currently observed rates, and there will be a *dual growth engine* at work: the R&D sector will produce labor-augmenting technological progress, augmenting in particular the cognitive tasks which will be immune to automation. The resulting productivity gains will then feed into the accumulation of R&D capital, improving R&D productivity and sustaining the constant rate of growth.

By contrast, if full automation turns out to be possible, either in production or R&D (or both), then we will observe a large, potentially order-of-magnitude acceleration in global economic growth (Davidson 2021; Hanson 2001). Once the essential cognitive tasks are fully automated, the pace of growth will be unpinned from the pace of growth in the effective (technology-augmented) supply of human cognitive work, and instead will follow the pace of accumulation of programmable hardware. The ultimate engine of growth will then no longer be labor-augmenting technical change but the accumulation of programmable hardware – computing power, digital memory and bandwidth.

Apart from these two possibilities, robustness checks uncover two additional scenarios which may appear *prima facie* equally reasonable – and have been discussed in some of the related literature – but, upon inspection, require assumptions that are likely at odds with the physical reality. In the third scenario, not only is full automation impossible but also there is no R&D capital in the production function for new ideas. In such a scenario the model produces the prediction of secular stagnation. However, this prediction rests on the problematic assumption that all cognitive *and physical* tasks in R&D must be carried out by people and there is no possibility of using any mechanical R&D equipment. Finally, in the fourth scenario a fraction of technological progress takes the hardware-augmenting form. In such a scenario the model produces explosive dynamics culminating in technological singularity. However, this prediction appears to require unbounded growth in energy efficiency, which contradicts the laws of thermodynamics.

The paper is related to studies focusing on automation and its impacts on productivity, employment, wages and factor shares. It has been argued in this literature that routine occupations in the middle of the wage distribution are the first to automate, resulting in polarization of the labor market (Acemoglu 2011; Autor and Dorn 2013), and that in the future automation is likely to gradually advance into less-and-less routine jobs (Arntz, Gregory, and Zierahn 2016; Frey and Osborne 2017). It has also been argued that automation raises average labor productivity while substituting low-skilled labor and complementing high-skilled labor (Graetz and Michaels 2018; Hemous and Olsen 2018). In turn, Andrews, Criscuolo, and Gal (2016), Autor et al. (2020), Barkai (2020) have documented a new trend, parallel to automation and the development of digital technologies – the emergence of a narrow group of best performing firms whose productivity grows much faster than all other firms, contributing to growing income inequality, a falling labor share of output, and a growing share of corporate profits. Moreover, influential studies such as Zeira (1998), Acemoglu and Restrepo (2018a, 2018b), Aghion, Jones, and Jones (2019) have put forward growth models allowing to quantify the impact of automation on long-run growth and factor shares. The current paper contributes to this literature by studying productivity growth, dominance of certain growth engines and the evolution of factor shares in the future under a number of stylized scenarios, using a relatively flexible framework which, in particular, allows automation to enter both production and R&D, and allows the labor share to grow or fall with automation, depending on whether full automation of essential cognitive tasks is possible or not.

The paper also refers to the fast growing literature on macroeconomic implications of development of "digital/robotic/machine labor", AI and autonomous robots. The first paper to study the potential long-run growth impacts of AI was due to Hanson (2001). Several studies have investigated the possible consequences of employing robots to perform tasks which have been hitherto performed by people (Berg, Buffie, and Zanna

2018; Benzell and Brynjolfsson 2019; Benzell et al. 2015; Caselli and Manning 2019; DeCanio 2016; Graetz and Michaels 2018; Sachs, Benzell, and LaGarda 2015). A big question which permeates this literature is in which circumstances automation, or otherwise the adoption of robots or AI, replaces or augments human work. Finally, a few studies have also investigated the potential of selected digital technologies, such as advanced AI, to potentially drive a growth acceleration or even a technological singularity (Davidson 2021; Trammell and Korinek 2021; Yudkowsky 2013). The current paper contributes to this literature by looking at the future through the lens of the *hardware–software framework* which sharply distinguishes between the physical and cognitive component of work, and has the notion of substitutability/complementarity between people and machines at its very core.

The remainder of the paper is structured as follows. Section 2 discusses the theoretical framework. Section 3 deals with the role of partial versus full automation. Section 4 covers the mixed cases where full automation is possible only in production or only in R&D. Section 5 discusses the role of R&D capital. Section 6 covers hardware-augmenting technical change. Section 7 summarizes the results and concludes.

2 Theoretical framework

2.1 Hardware and software as factors of production

The key observation underlying the hardware–software framework (Growiec 2019) is that all output is generated through purposefully initiated physical action. Generating output requires both some physical action involving energy – carried out by *hardware* – and some information describing the action – provided by *software*.[1] This underscores that physical capital and human physical labor are fundamentally substitutable inputs, contributing to *hardware*; analogously, human cognitive work and pre-programmed digital software are also substitutes, contributing to *software*. In turn, both hardware and software are complementary and essential in the process. Furthermore, programmable hardware, such as computers, smartphones or robots, similarly to the human body has double duty: as means of performing physical action and as a container for software – stored information and working algorithms.

In other words, producing output requires both some physical *action* and some *code*, a set of instructions describing and purposefully initiating the action. Based on this premise I posit a general production function (for whatever output) featuring some physical *hardware* X, able to perform the action, and some disembodied *software* S, providing the information:

$$\text{Output} = \mathcal{F}(X, S). \tag{1}$$

I assume furthermore that $\mathcal{F} : \mathbb{R}^2_+ \to \mathbb{R}_+$ is increasing and concave in both factors and such that hardware X and software S are essential (i.e., $\mathcal{F}(0, S) = \mathcal{F}(X, 0) = 0$) and mutually complementary in production (the elasticity of substitution between X and S is below unity). One natural way to instantiate this assumption is to take a constant elasticity of substitution (CES) specification with an elasticity of substitution $\sigma \in (0, 1)$, cf. Klump, McAdam, and Willman (2007, 2012). The particular CES form of the $\mathcal{F}$ function is however not necessary for the results.

Hardware X ("brawn") includes physical actions performed by both humans and machines. Hence, X encompasses both the services of physical capital K and human physical labor L, where the latter variable excludes any know-how or skill of the worker.

Software S ("brains"), in turn, encompasses all useful instructions which stem from the available information, in particular the practical implementation of state-of-the-art technologies. Hence, it includes the skills and technological knowledge employed in human cognitive work, H, as well as pre-programmed

1 This approach is well grounded in physics. As a frank summary, Michio Kaku said: "I'm a physicist. We rank things by two parameters: energy and information".

software Ψ, which is essentially a task-specific list of instructions to be performed by the associated programmable hardware (e.g., computers, robots, smartphones, etc.). Pre-programmed software Ψ may in particular include artificial intelligence (AI) algorithms, able to learn from data as well as potentially self-improve and self-replicate.

Within hardware X, capital and labor are inessential and substitutable as agents of physical action (elasticity of substitution above unity). This reflects the idea that whatever performs a given set of actions, if the actions are the same then the outcome should be the same, too. The same logic applies to software S: regardless of whether a set of instructions comes from a human brain or a mechanical information processing unit, if the actual information content of instructions is the same, then the outcome should be the same, too. An important caveat for the case of software, though, is that we don't know yet if all types of instructions can be provided by both people and machines, that is whether all essential tasks can be potentially automated in the future. As it turns out, if certain essential cognitive tasks will never be automated, then the reduced form production function will feature complementarity of human cognitive work and pre-programmed software within software S (elasticity of substitution below unity).[2]

Formally, I represent these assumptions as:

$$X = G_1(L, K), \quad S = G_2(H, \Psi), \tag{2}$$

where the elasticity of substitution in G_1 is above one, and in G_2 – above one in the full automation scenario, and below one in the partial automation scenario. The functions $G_1 : \mathbb{R}_+^2 \to \mathbb{R}_+$ and $G_2 : \mathbb{R}_+^2 \to \mathbb{R}_+$ are assumed increasing and concave in both inputs. The replacement of L with K within hardware X will be referred to as *mechanization*, whereas the replacement of H with Ψ within software S will be called *automation*.

Each of the four factors of production has its unique properties.

- *Human physical labor* L is rivalrous and given in fixed supply per worker and unit of time, $L = \zeta N$ where $\zeta \in [0, \bar{\zeta}]$ denotes the supply of physical labor per worker in a unit of time, expressed in physical capital units, and N is the total number of workers.
- *Physical capital* K is rivalrous but can be unboundedly accumulated in per-capita terms. Physical capital K may be non-programmable or programmable. The share of programmable hardware in total physical capital is denoted by χ (so that $\chi \in [0, 1]$).
- *Human cognitive work* H consists of three components: technological knowledge A, the average skill level h, and the number of workers N, as in $H = AhN$. Technological knowledge A, also interpreted as the size of the repository of task-specific codes, is non-rivalrous (Romer 1986, 1990) and accumulable. Per-capita skill levels h are rivalrous and bounded above.
- *Pre-programmed software* Ψ also consists of three components: technological knowledge A, algorithmic skill level ψ which captures the degree to which pre-programmed software is able to perform the tasks collected in A, and the stock of programmable hardware χK on which the software is run, as in $\Psi = A\psi\chi K$. Technological knowledge A is the same as above.[3] The algorithmic skill level ψ is assumed to be bounded above by the optimal code for performing a given task (i.e., perfect accuracy), though there may be in fact a much lower upper bound $\bar{\psi}$ (Hanson and Yudkowsky 2013). Because digital software can be virtually costlessly copied, it is assumed that it can scale up to the level of available programmable hardware χK.

All in all, the general production function takes the form:

$$\text{Output} = \mathcal{F}(G_1(\zeta N, K), G_2(AhN, A\psi\chi K)). \tag{3}$$

2 The issue of partial versus full task automatability and its mapping to the elasticity of substitution is discussed at length in Growiec (2022b). The key conclusions are summarized in Section 2.3 below.

3 If in reality the sets of codes available to humans and digital algorithms are different, the discrepancy between the measures of both sets can be captured by the ratio ψ/h.

Finally, following Romer (1986, 1990) the hardware–software model envisages technological progress (growth in A), produced by the R&D sector, as expansion of the "repository of codes", i.e., as the development of new, better instructions allowing to produce higher output with given hardware. Whether these new instructions take the form of new abstract ideas, scientific theories, systematically catalogued facts, codes specifying certain actions, or blueprints of physical items, they are all *information* and not actual *objects* or *actions*, and it is precisely this informational character that makes technologies non-rivalrous and a source of increasing returns to scale (Romer 1990). In contrast to Paul Romer's seminal contributions, though, here these instructions can be applied to the tasks at hand both by humans and machines. Thus all technological progress is naturally modeled as *software-augmenting*.

2.2 Reduced form economic growth model

In the current paper I apply the hardware–software framework to two distinct economic activities: *production* and *R&D*. The output of production is the final good Y, serving both consumption and investment purposes, whereas R&D produces new technological ideas $\dot{A}$. I extend the original framework of Growiec (2019) to accommodate a range of alternative assumptions regarding automatability of production and R&D. As additional robustness checks I also discuss a few alternative specifications of the R&D process.

The following reduced form two-sector economic growth model with a production and R&D sector serves as the baseline:

$$Y = F(G_1(\zeta N, K), G_2(AhN, A\psi\chi K)), \tag{4}$$

$$\dot{A} = A^{\phi}\Phi(G_1(\zeta N, K), G_2(AhN, A\psi\chi K)), \tag{5}$$

$$\dot{K} = sY - \delta K, \tag{6}$$

where the term A^{ϕ} (with $\phi \in [0,1]$) captures the potentially positive "standing on shoulders" effects in R&D (Jones 1995). The aggregate production function $F : \mathbb{R}^2_+ \to \mathbb{R}_+$ and the idea production function $\Phi : \mathbb{R}^2_+ \to \mathbb{R}_+$ are assumed to have the properties of the general production function $\mathcal{F}$ discussed above.[4] I additionally assume that F, G_1, G_2 and Φ are characterized by constant returns to scale.

Finally, I posit that bounded variables (s, h, ψ, χ) will eventually stabilize, and so will global human population N. In my derivations I concentrate solely on the dynamics of two state variables of the model, K and A, in the long-run limit, treating s, h, ψ, χ and N as given constants, just like in the canonical Solow model (cf. Jones 2005; Solow 1956).

In the following analysis I use the following approximations as $K/N \to \infty$:

$$X \approx \alpha K \quad \text{where} \quad \alpha = G_1(0,1) = \lim_{x\to 0} G_1(x,1), \tag{7}$$

$$S \approx \beta A\psi\chi K \quad \text{where} \quad \beta = G_2(0,1) = \lim_{x\to 0} G_2(x,1), \quad \text{full automation}, \tag{8}$$

$$S \approx \gamma AhN \quad \text{where} \quad \gamma = G_2(1,\infty) = \lim_{y\to\infty} G_2(1,y), \quad \text{partial automation}. \tag{9}$$

These approximations are valid thanks to the assumption of constant returns to scale as well as above unitary elasticity of substitution in G_1 (Eq. (7)) and in G_2 (Eq. (8)), and alternatively, below unitary elasticity of substitution in G_2 (Eq. (9)). They are derived by computing the limits of appropriate ratios. With regard to Eqs.

4 Using a CES ideas production function specification, Growiec, McAdam, and Mućk (2022) demonstrate empirically for the US that capital and labor are gross complements also in R&D ($\sigma_{\text{R\&D}} < 1$).

(7)–(9) I obtain, respectively:[5]

$$\lim_{\frac{K}{N}\to\infty}\frac{X}{K} = \lim_{\frac{K}{N}\to\infty}\frac{G_1(\zeta N, K)}{K} = \lim_{\frac{K}{N}\to\infty} G_1\left(\zeta\frac{N}{K}, 1\right) = G_1(0,1) = \alpha > 0, \tag{10}$$

$$\lim_{\frac{K}{N}\to\infty}\frac{S}{K} = \lim_{\frac{K}{N}\to\infty} A\psi\chi G_2\left(\frac{hN}{\psi\chi K}, 1\right) = A\psi\chi G_2(0,1) = \beta A\psi\chi > 0, \text{ full automation}, \tag{11}$$

$$\lim_{\frac{K}{N}\to\infty}\frac{S}{N} = \lim_{\frac{K}{N}\to\infty} AhG_2\left(1, \frac{\psi\chi K}{hN}\right) = AhG_2(1,\infty) = \gamma Ah > 0, \text{ partial automation}. \tag{12}$$

I also use the following asymptotic notation:

$$a_K = F(1,\infty) = \lim_{y\to\infty} F(1,y), \quad b_K = \Phi(1,\infty) = \lim_{y\to\infty}\Phi(1,y), \tag{13}$$

$$a_N = F(\infty,1) = \lim_{x\to\infty} F(x,1), \quad b_N = \Phi(\infty,1) = \lim_{x\to\infty}\Phi(x,1). \tag{14}$$

By the assumptions of constant returns to scale and less than unitary elasticity of substitution in F and Φ, limits in (13) and (14) exist and are finite.

2.3 Link to a task-based framework

The model discussed above does not explicitly consider tasks. However, it can be viewed as a reduced form representation of a richer task-based framework that includes specifically physical tasks (aggregated within hardware) and cognitive tasks (within software). Replacement of labor with capital in physical tasks would then be understood as mechanization, and in cognitive tasks – as automation.

Let me now focus on automation, a key trend of the digital era. Like in Growiec (2022b), one could consider cognitive tasks consisting of a number of essential and complementary sub-tasks. Without loss of generality, we can say there are two such sub-tasks. Then two alternative scenarios can be compared: (i) *partial automation*, where sub-task 2 is not automatable and can be performed by humans only, and (ii) *full automation*, where both sub-tasks can be performed both by humans and programmable machines.

Under partial automation, two regimes emerge in equilibrium (Growiec 2022b): if the effective supply of programmable machines relative to human cognitive work (in the current notation, $\frac{A\psi\chi K}{AhN}$) is low, then human work is used in both sub-tasks, and if it is high – only in the non-automatable sub-task 2. By assumption, machines are always used only in sub-task 1. In turn, under full automation there are three regimes in equilibrium: if $\frac{A\psi\chi K}{AhN}$ is low, then human work is used in both sub-tasks and machines are used only in sub-task 1. For intermediate values of $\frac{A\psi\chi K}{AhN}$ there is perfect specialization, so that sub-task 1 employs only machines, and sub-task 2 employs only people. Finally, if $\frac{A\psi\chi K}{AhN}$ is high, human work is used only in sub-task 2 and machines are employed in both sub-tasks, i.e., both sub-tasks get automated.

In the partial automation scenario, as $\frac{A\psi\chi K}{AhN}$ grows the aggregate elasticity of substitution (Miyagiwa and Papageorgiou 2007; Xue and Yip 2013) between human cognitive work and programmable machines declines from a high value $\sigma_{HI} > 1$ when human cognitive work is relatively abundant, to a low value $\sigma_{LO} < 1$

5 To take an illustrative example, let G_1 take the CES form with an elasticity of substitution σ above one (so that $\theta = \frac{\sigma-1}{\sigma} \in (0,1)$),

$$X = G_1(\zeta N, K) = \left(\pi(\zeta N)^\theta + (1-\pi)K^\theta\right)^{\frac{1}{\theta}}, \quad \pi, \theta \in (0,1).$$

Under this assumption α can be computed explicitly in terms of the parameters of the production function:

$$\alpha = \lim_{\frac{K}{N}\to\infty}\frac{X}{K} = \lim_{\frac{K}{N}\to\infty}\left(\pi\left(\frac{\zeta N}{K}\right)^\theta + (1-\pi)\right)^{\frac{1}{\theta}} = (1-\pi)^{\frac{1}{\theta}}.$$

when it is relatively scarce. Then the supply of human cognitive work becomes a bottleneck for economic growth. Specifically as $\frac{A\psi\chi K}{AhN} \to \infty$, the aggregate elasticity of substitution converges to the low elasticity of substitution between the two complementary sub-tasks and the human and machine inputs are gross complements.

In contrast, in the full automation scenario, as $\frac{A\psi\chi K}{AhN}$ grows the aggregate elasticity of substitution between human cognitive work and programmable machines first declines from a high value $\sigma_{\mathrm{HI}} > 1$ when human cognitive work is relatively abundant, to $\sigma_{\mathrm{LO}} < 1$ when machines specialize in sub-task 1 and people in sub-task 2, but then rises again to $\sigma_{\mathrm{HI}} > 1$ as both tasks get automated. Specifically as $\frac{A\psi\chi K}{AhN} \to \infty$, the aggregate elasticity of substitution converges to the high elasticity of substitution between human cognitive work and programmable machines within the two sub-tasks, so that the human and machine inputs are gross substitutes.

All in all, this task-based model of automation lends support for the use of above unitary elasticity of substitution in the reduced form representation G_2 (Eq. (8)) in the full automation case, and below unitary elasticity of substitution in G_2 (Eq. (9)) in the partial automation case, as $K/N \to \infty$ (with constant ψ, χ and h). Furthermore, under the assumption that all *physical* tasks can potentially be performed by machines ("full mechanization"), it also lends support to the assumption of above unitary elasticity of substitution in G_1 (Eq. (7)) as $K/N \to \infty$.

Note, though, an important distinction between the aforementioned task-based model, set within the hardware–software framework, and models which view automation as an expansion of the set of tasks that can be produced with capital instead of labor (such as Acemoglu and Restrepo, 2018a, 2018b; Zeira 1998). The two-level nested production function used here makes a big difference. For example, Acemoglu and Restrepo (2018a) criticize the literature which models automation as *capital-augmenting technical change* on the premise that these studies imply that automation reduces the labor share only if the elasticity of substitution between capital and labor is greater than one (gross substitutability), whereas in reality we observe automation, falling labor shares and gross complementarity. This criticism is valid but does not apply here: in the hardware–software framework automation is understood as replacement of people with machines within software, occurring either because of the accumulation of programmable hardware or increases in the algorithmic skill level ψ. The impact of automation on the labor share is generally ambiguous, but specifically automation will reduce the labor share if the supply of programmable machines is sufficiently low (as it was in historical data), or if it is high and we are approaching full automation. And all this may well happen despite aggregate capital and labor being gross complements, which is theoretically predicted from the hardware–software model if capital is used predominantly within hardware, and labor – within software.

2.4 Overview of results

The analysis produces the following results.

- **Full versus Partial Automation**. *How is the long-run growth rate and its driving force affected whether or not all essential tasks can be automated?*
  I find that when all essential production and R&D tasks are subject to automation, the long-run growth engine, determining the GDP growth rate in the long-run limit, is the accumulation of programmable hardware. This scenario is consistent with a (potentially massive) growth acceleration in the future. If some essential tasks cannot be automated, though, both in production and R&D, a dual long-run growth engine emerges, consisting of R&D (generating labor-augmenting developments) and the accumulation of programmable hardware (used as R&D capital). Then the GDP growth rate in the long-run limit is constrained by the rate of R&D which uses the essential and scarce human input.
- **Automation of Production versus R&D**. *How is the long-run growth rate and its driving force affected if only production, but not R&D tasks can be automated? And conversely, what if only R&D, but not production tasks can be automated?*

I find that if all essential production tasks are subject to automation, the long-run growth engine, determining the GDP growth rate in the long-run limit, is the accumulation of programmable hardware. Whether or not R&D tasks can be automated as well, is irrelevant for long-run growth. Analogously, if all essential R&D tasks can be automated, automation of R&D has the potential of accelerating and sustaining long-run growth by creating a positive feedback loop in the R&D sector. Then the long-run growth engine, determining the GDP growth rate in the long-run limit, is again the accumulation of programmable hardware, and whether some essential production tasks may not be automated, becomes irrelevant for the long-run growth rate. All these scenarios are consistent with a substantial growth acceleration in the future.

– **R&D Capital**. *How is the long-run growth rate and its driving force affected whether or not machines (physical capital) can be used in the R&D process?*
  I find that when all essential production tasks are subject to automation, presence or absence of R&D capital in the R&D process is irrelevant for the GDP growth rate in the long-run limit, which is determined by the pace of accumulation of programmable hardware. If some essential production tasks cannot be automated, though, accumulation of R&D capital is necessary for sustaining long-run growth via a positive feedback loop in the R&D sector. A dual long-run growth engine emerges then, consisting of R&D (generating labor-augmenting developments) and the accumulation of programmable hardware (used as R&D capital). Otherwise, if neither production nor R&D tasks can be fully automated and R&D capital is not used in the R&D process, the GDP growth rate in the long-run limit is driven exclusively by the R&D sector producing labor-augmenting technical change. With R&D output being critically constrained by the supply of the scarce human input, this scenario predicts declining GDP growth rates over the long run (secular stagnation).
– **Hardware-Augmenting Technical Change**. *How is the long-run growth rate and its driving force affected whether or not technical change can be (at least partly) hardware-augmenting?*
  I find that when all essential production or R&D tasks are subject to automation, hardware-augmenting technical change leads to explosive growth with unboundedly increasing GDP growth rates (technological singularity). This occurs due to the creation of a self-reinforcing positive feedback loop between hardware accumulation and hardware-augmenting technical change. If some essential tasks both in production and R&D cannot be automated, though, hardware-augmenting technical change matters only insofar as it accelerates the R&D process itself.

These results are intuitive. The key variable to observe is the relatively scarce factor of production in the long-run limit. Is it hardware or software?

In the pre-1980 industrial economy, as production processes were increasingly mechanized but not automated (capital was gradually replacing human physical labor in performing physical actions, i.e. in *hardware*, but instructions for the actions were provided exclusively by people), the scarce factor was human cognitive work, which is not accumulable per capita. Then the key source of long-run growth was labor-augmenting technological progress, provided by R&D (Acemoglu 2009; Jones 1995; Romer 1990).

In the post-1980 economy, though, following the dawn of the digital era production processes increasingly get automated (instructions for physical actions are increasingly stored and run on programmable hardware). Labor is replaced with capital in *software*. By increasing the supply of the scarce software factor, automation contributes to economic growth alongside labor-augmenting R&D. As of 2021, R&D remains the key growth engine among the two because there is a wide range of tasks which – with today's technology – cannot be automated. However, if eventually all essential tasks will be automated, labor (specifically, human cognitive work) will give way to capital (specifically, programmable hardware) as the scarce factor of production. Then the key source of growth will be the accumulation of programmable hardware (Growiec 2019; Jones and Manuelli 1990). Such a scenario is associated with a potentially massive acceleration in the GDP growth rate: following the digital revolution, growth in data volume, processing power and bandwidth is an order of magnitude faster than growth in global GDP (Hilbert and López 2011).

If, in contrast, some essential production and R&D tasks will never be automated, human labor employed in these tasks will remain the scarce factor of production, and labor-augmenting technological progress which improves its productivity will remain the key engine of long-run growth. The pace of GDP growth will then remain tied to the dynamics of R&D output, sustained by the accumulation of R&D capital (hardware in the R&D sector) but constrained by the effective supply of R&D labor.

However, if one excluded the possibility of systematic accumulation of R&D capital, able to sustain constant growth in R&D productivity – like in e.g. Romer (1990), Jones (1995, 2002), Bloom et al. (2020) – then the dynamics of R&D output would be most likely subject to a secular slowdown or stagnation in the coming decades, and so will aggregate output in the economy.

On top of all that, the hypothetical force of hardware-augmenting technical change – understood e.g. as increases in energy efficiency, particularly of computers and other programmable machines – can alleviate the scarcity of programmable hardware. Its effects will therefore be particularly notable in the scenarios where hardware really is relatively scarce, i.e., in the scenarios where production and/or R&D can be fully automated.[6] This possibility lends some justification for the hypothesis of an upcoming technological singularity, but should be treated with caution because unbounded increases in energy efficiency are at odds with laws of thermodynamics.

3 Full versus partial automation

Let us begin the study of prospective sources of global economic growth across a range of scenarios for the digital era. I first observe that the answer depends crucially on whether all essential tasks may be potentially automated in the future (the *full automation* case), or some of them will always have to be performed by humans (the *partial automation* case).

Full Automation of Production and R&D. In the long-run limit, $K/N \to \infty$ and all tasks eventually get mechanized and automated. Then for computing the long-run dynamics we may approximate $X = G_1(\zeta N, K) \approx \alpha K$ and $S = G_2(AhN, A\psi\chi K) \approx \beta A\psi\chi K$. As $K \to \infty$ and $A \to \infty$, asymptotically

$$Y = F(\alpha K, \beta A\psi\chi K) = \alpha K F(1, A\psi\chi \cdot \beta/\alpha) \to \alpha a_K K, \tag{15}$$

$$\dot{A} = A^{\phi}\Phi(\alpha K, \beta A\psi\chi K) = A^{\phi}\alpha K\Phi(1, A\psi\chi \cdot \beta/\alpha) \to \alpha b_K A^{\phi} K, \tag{16}$$

$$\dot{K} \approx (s\alpha a_K - \delta)K. \tag{17}$$

This means that when all essential tasks are subject to automation, the GDP growth rate will converge to $g = g_K = s\alpha a_K - \delta$, and the long-run growth engine will be the accumulation of programmable hardware (Jones and Manuelli 1990). Because hardware is accumulated in proportion to K, whereas software – in proportion to AK, the stock of software will grow systematically faster than hardware, and therefore (programmable) hardware will eventually become the scarce factor of production. The pace of technical change (growth in A) will eventually become irrelevant for growth as the latter will de-couple from the bounded supply of human inputs.

Partial or No Automation of Production and R&D. In the long-run limit, all tasks will eventually get mechanized while a fraction of essential production and R&D tasks will forever remain immune to automation, making human cognitive work and pre-programmed software complementary (elasticity of substitution below one in G_2). As $K/N \to \infty$, for computing the long-run dynamics we may approximate $X = G_1(\zeta N, K) \approx \alpha K$ and $S = G_2(AhN, A\psi\chi K) \approx \gamma AhN$.

6 Possibilities and limits to future growth in energy efficiency have been reviewed by Beaudreau and Lightfoot (2015).

There are two sub-cases to consider here, either $\phi = 0$ or $\phi \in (0, 1]$. If $\phi = 0$ (no "standing on shoulders" effects) then an asymptotic balanced growth path is attained as $K \to \infty$ and $A \to \infty$, with K/A approaching a constant and

$$g = g_K = g_A = \Phi\left(\alpha \frac{K}{A}, \gamma hN\right) = sF\left(\alpha, \gamma \frac{A}{K} hN\right) - \delta. \tag{18}$$

The formula (18) is a system of two equations in two variables, g and K/A, that allows a unique solution.

In this case, the long-run GDP growth rate is determined by the dynamics of R&D output, which are in turn sustained by the accumulation of R&D capital. This is a *dual growth engine*, and both hardware and software grow asymptotically at the same rate g, limited and determined by the scarce supply of human cognitive work hN.

In contrast, with positive R&D spillovers ("standing on shoulders", Jones 1995), asymptotically A will grow faster than K. When $\phi \in (0, 1]$, in the long-run limit

$$Y = F(\alpha K, \gamma AhN) = \alpha KF\left(1, \frac{\gamma A}{\alpha K} hN\right) \to \alpha a_K K, \tag{19}$$

$$\dot{A} = A^{\phi}\Phi(\alpha K, \gamma AhN) = A^{\phi}\alpha K\Phi\left(1, \frac{\gamma A}{\alpha K} hN\right) \to \alpha b_K A^{\phi} K, \tag{20}$$

$$\dot{K} \approx (s\alpha a_K - \delta)K. \tag{21}$$

Hence, in the presence of R&D capital accumulation, positive R&D spillovers reinstate the Jones and Manuelli (1990) dynamic even if production and R&D tasks are only partially automatable or not at all. This is because positive R&D spillovers are a multiplicative factor in the R&D equation and thus also *partially augment the hardware factor* employed in this sector, including accumulable R&D capital. The long-run growth engine is then again the accumulation of programmable hardware, setting the GDP growth rate in the long-run limit as $g = g_K = s\alpha a_K - \delta$, i.e., in particular de-coupling it from the supply of the scarce human input. However, the fact that $\phi > 0$ implies the existence of hardware-augmenting technical change in R&D makes this case arguably less realistic than $\phi = 0$, and therefore in the discussion I consider $\phi = 0$ as the baseline.

4 Full automation only in production or R&D

Literature suggests that routine tasks, both manual and cognitive, are relatively easiest to automate, while automation gets harder for tasks which are more complex and carried out in a less structured environment. Among all tasks, cutting-edge R&D tasks requiring sophisticated reasoning and out-of-the-box thinking are probably among the least susceptible to automation (Acemoglu 2011; Autor and Dorn 2013; Frey and Osborne 2017). It is therefore natural to expect that production tasks may – if at all – become fully automatable earlier than R&D tasks.[7] In order not to miss any viable scenario of the future, however, in the following paragraphs I discuss both the scenario in which production eventually becomes fully automatable, whereas R&D does not (in line with the assumptions made by, among many others, Acemoglu and Restrepo 2018b),[8] and the opposite scenario where eventually R&D becomes fully automatable, whereas production does not.

Full automation only in production. In the long-run limit, as all production tasks eventually get mechanized and automated, while human cognitive work remains essential for R&D, asymptotically (with $K \to \infty$ and $A \to \infty$) I obtain:

$$Y \approx F(\alpha K, \beta A\psi\chi K) = \alpha KF(1, A\psi\chi \cdot \beta/\alpha) \to \alpha a_K K, \tag{22}$$

7 This said, we also see that AI algorithms are already entering research tasks, such as scanning astronomical photographs, sequencing genomes, or predicting patterns of protein folding (AlphaFold), while some seemingly easy motor tasks remain notoriously difficult to automate – so perhaps we should be cautious with such "natural" predictions.

8 As explained in Figure 2 of Acemoglu and Restrepo (2018b) and the surrounding text, in their model all newly invented tasks are assumed to be initially produced with labor. Moreover, R&D itself also uses only the labor of scientists (Eq. (22) in Acemoglu and Restrepo (2018b)).

$$\dot{A} \approx A^{\phi}\Phi(\alpha K, \gamma AhN), \tag{23}$$

$$\dot{K} \approx (s\alpha a_K - \delta)K. \tag{24}$$

Hence, once all essential production tasks are automated, the long-run GDP growth rate will converge to $g = g_K = s\alpha a_K - \delta$, and the long-run growth engine will be the accumulation of programmable hardware (Jones and Manuelli 1990). Because hardware is accumulated in proportion to K, whereas pre-programmed software used in production – in proportion to AK, the stock of production software will grow systematically faster than hardware, and therefore (programmable) hardware will eventually become the scarce factor of production.[9]

Full automation only in R&D. In the long-run limit, as all R&D tasks will eventually get mechanized and automated, while human cognitive work will remain essential for production, asymptotically A will be growing faster than K, implying (as $A \to \infty$ and $K \to \infty$):

$$Y \approx F(\alpha K, \gamma AhN) = \alpha KF\left(1, \frac{\gamma A}{\alpha K}hN\right) \to \alpha a_K K, \tag{25}$$

$$\dot{A} \approx A^{\phi}\Phi(\alpha K, \beta A\psi\chi K) = A^{\phi}\alpha K\Phi\left(1, A\psi\chi \cdot \beta/\alpha\right) \to \alpha b_K A^{\phi} K, \tag{26}$$

$$\dot{K} = (s\alpha a_K - \delta)K. \tag{27}$$

It turns out that full automation of R&D tasks is sufficient for generating the Jones and Manuelli (1990) dynamic with a long-run GDP growth rate $g = g_K = s\alpha a_K - \delta$ even if production tasks are only partially automatable or not at all. The long-run growth engine is then again the accumulation of programmable hardware. Compared to the scenario where neither production nor R&D tasks are fully automatable, full automation of R&D creates an additional positive feedback loop, accelerating and sustaining long-run growth. Compared to the scenario with full automation in production and R&D, though, the GDP growth rate is probably markedly lower over the transition and only converges to the same rate in the long-run limit.

5 R&D capital

Nowadays R&D processes increasingly use sophisticated machinery. 21st century science would not be possible without specialized lab equipment, not to mention the general computing capacity provided by personal computers on researchers' laps. Economic growth theory thus far has however rarely acknowledged this fact, focusing almost exclusively on the other crucial R&D input – researchers' skilled work.[10] Hence, to bring the current paper closer to the established R&D-based growth literature (Acemoglu 2009; Barro and Sala-i-Martin 2003; Bloom et al. 2020; Jones 1995; Romer 1990), I will now ask the question if the long-run predictions from the hardware–software framework would be affected if the role of R&D capital was disregarded. Therefore in the following paragraphs, as a robustness check, I consider a version of my reduced form economic growth model without R&D capital, i.e., without ever allowing R&D processes to be mechanized. To this end I fix $X = \zeta N$ in the R&D sector. I separately discuss the cases of partial versus full automation in production and R&D.

9 Comparing Eqs. (16) and (23), one could hypothesize that along the transition, the pace of technical change (growth in A) will be probably markedly lower here than in the scenario where R&D tasks are automated as well. This should drag also on GDP growth over the transition. In the long-run limit, though, the role of R&D will vanish and its pace will eventually become irrelevant for the pace of GDP growth.

10 Even the seminal "lab equipment" specification of the R&D sector (Rivera-Batiz and Romer 1991) includes the capital input only indirectly.

No R&D capital, full automation in production. In the long-run limit, as all production tasks eventually get mechanized and automated ($X \approx \alpha K, S \approx \beta A\psi\chi K$) while human physical work remains essential for R&D tasks ($X = \zeta N, S \approx \beta A\psi\chi K$), asymptotically I obtain as $A \to \infty$ and $K \to \infty$:

$$Y = F(\alpha K, \beta A\psi\chi K) = \alpha KF(1, A\psi\chi \cdot \beta/\alpha) \to \alpha a_K K, \tag{28}$$

$$\dot{A} = A^{\phi}\Phi(\zeta N, \beta A\psi\chi K) = A^{\phi}\zeta N\Phi\left(1, A\frac{\beta\psi\chi K}{\zeta N}\right) \to b_K A^{\phi}\zeta N, \tag{29}$$

$$\dot{K} \approx (s\alpha a_K - \delta)K. \tag{30}$$

Alternatively, with partial or no automation of R&D tasks ($S \approx \gamma AhN$ in R&D), the R&D equation becomes $\dot{A} = A^{\phi}\Phi(\zeta N, \gamma AhN) = A^{\phi}\zeta N\Phi\left(1, A\frac{\gamma h}{\zeta}\right) \to b_K A^{\phi}\zeta N$, with exactly the same asymptotic result for the GDP growth rate.

I observe that with full automation of production tasks, the accumulation of programmable hardware is the key growth engine over the long run, and the GDP growth rate converges to $g = g_K = s\alpha a_K - \delta$ in the limit. Under full automation of production, the presence or absence of R&D capital in the R&D process is irrelevant for the asymptotic results.

No R&D capital, full automation only in R&D. In the long-run limit, as all production tasks eventually get mechanized but not automated ($X \approx \alpha K, S \approx \gamma AhN$) while human physical but not cognitive work remains essential for R&D tasks ($X = \zeta N, S \approx \beta A\psi\chi K$), asymptotically (as $A \to \infty, K \to \infty$) technology A grows at a faster rate than K, implying:

$$Y = F(\alpha K, \gamma AhN) = \alpha KF\left(1, \frac{\gamma A}{\alpha K}hN\right) \to \alpha a_K K, \tag{31}$$

$$\dot{A} = A^{\phi}\Phi(\zeta N, \beta A\psi\chi K) = A^{\phi}\zeta N\Phi\left(1, A\frac{\beta\psi\chi K}{\zeta N}\right) \to b_K A^{\phi}\zeta N, \tag{32}$$

$$\dot{K} \approx (s\alpha a_K - \delta)K. \tag{33}$$

It turns out that full automation of R&D tasks suffices to make accumulation of programmable hardware the key growth engine over the long run even if all physical R&D tasks are carried out by people. Asymptotically, the economy follows the Jones and Manuelli (1990) dynamic and the GDP growth rate converges to $g = g_K = s\alpha a_K - \delta$. In the short to medium run, though, the failure to fully automate production processes most probably provides a major drag on the pace of growth.

No R&D capital, partial or no automation. In the long-run limit, as all tasks eventually get mechanized but a fraction of essential production and R&D tasks is immune to automation, for computing the long-run dynamics (where $K/N \to \infty$) we may approximate $X \approx \alpha K$ in production and $S \approx \gamma AhN$ in production and R&D. For the latter sector we consequently obtain as $A \to \infty$ and $K \to \infty$:

$$\dot{A} = A^{\phi}\Phi(\zeta N, \gamma AhN) = A^{\phi}\zeta N\Phi\left(1, A\frac{\gamma h}{\zeta}\right) \to b_K A^{\phi}\zeta N. \tag{34}$$

Hence, with constant population N technology progresses sub-exponentially if $\phi \in [0, 1)$ (Groth, Koch, and Steger 2010; Jones 1995), or exponentially if $\phi = 1$ (Romer 1990). In the absence of R&D capital and with partial or no automation, the hardware–software framework reproduces the well known scenarios of R&D based growth in the industrial era: either semi-endogenous ($\phi < 1$, Jones) or fully endogenous growth ($\phi = 1$, Romer). The ultimate source of growth, determining the long-run GDP growth rate, is R&D. Software (in this case synonymous with human cognitive work) forever remains the scarce factor of production, limiting the pace of economic growth.

Specifically in the linear case $\phi = 1$, the GDP growth rate converges asymptotically to:

$$g = g_A = g_K = b_K\zeta N, \tag{35}$$

and thus is proportional to the "weakest link" in the economy, unaugmentable physical labor employed in R&D. If $\phi < 1$, without population growth the rate of technological progress g_A, and consequently the GDP growth rate g, is bound to systematically slow down over time (Jones 1995), in line with the secular stagnation hypothesis.

6 Hardware-augmenting technical change

In Growiec (2019) I have argued, grounding my points in Romer's seminal contributions, that technical change should be generally modeled as software-augmenting. After all, technological progress (growth in A) represents expansions of the "repository of codes", i.e., the development of new, better instructions allowing to produce higher output with given hardware. These instructions are *information* and not actual objects or actions, and it is precisely this informational character that makes technologies non-rivalrous and a source of increasing returns to scale (Romer 1986, 1990).

Hardware, by contrast, performs physical actions which require expediting energy. With this in mind, against the spirit of Romer's theory one could tentatively conjecture that certain improvements in energy efficiency of physical actions could potentially count as hardware-augmenting technical change. In the following paragraphs I will entertain this possibility as a second robustness check of my baseline results.

I find that hardware-augmenting technical change, *if unbounded*,[11] has the potential to resolve the scarcity of programmable hardware in production over the long run limit, thereby accelerating growth beyond the limits set by the constant-returns-to-scale, asymptotically linear character of the aggregate production function. When accumulation of programmable hardware is accompanied with R&D which is at least partly hardware-augmenting, these two forces have the potential of mutual reinforcement, creating self-reinforcing feedback loops that lead to explosive, super-exponential growth.

Technically, the key modification of the framework is that the hardware factor in production and R&D is now technologically augmented: $X = A^\kappa G_1(\zeta N, K)$, with $\kappa \in (0,1)$ representing the assumption that technological progress is partly hardware-augmenting but nevertheless remains biased towards software. Furthermore, in my baseline case I will ignore possible positive R&D spillovers ("standing on shoulders" effects) by setting $\phi = 0$. The remaining assumptions remain in place. The results are as follows.

Hardware-augmenting technical change with full automation. In the long-run limit, as all tasks will eventually get mechanized and automated, for computing the long-run dynamics we may approximate $X \approx A^\kappa \alpha K$ and $S \approx \beta A \psi \chi K$. As $A \to \infty$ and $K \to \infty$, asymptotically:

$$Y = F(A^\kappa \alpha K, \beta A \psi \chi K) = A^\kappa \alpha K F(1, A^{1-\kappa} \psi \chi \cdot \beta/\alpha) \to \alpha a_K A^\kappa K, \tag{36}$$

$$\dot{A} = \Phi(A^\kappa \alpha K, \beta A \psi \chi K) = A^\kappa \alpha K \Phi(1, A^{1-\kappa} \psi \chi \cdot \beta/\alpha) \to \alpha b_K A^\kappa K, \tag{37}$$

$$\dot{K} \approx (s \alpha a_K A^\kappa - \delta) K. \tag{38}$$

Hence, when all essential production tasks are subject to automation and technical change is partly hardware-augmenting, the long-run GDP growth rate $g = s\alpha a_K A^\kappa - \delta$ is ever increasing over time. The self-reinforcing dual long-run growth engine – the accumulation of programmable hardware plus hardware-augmenting technical change – generates explosive growth with an unbounded growth rate.

11 Unbounded growth in energy efficiency of material processes is physically impossible as it contradicts the laws of thermodynamics (see e.g. Beaudreau and Lightfoot 2015). Bounded improvements are possible, though. In particular, one could reasonably argue that the inventions of the Industrial Revolution which provided access to new efficient sources of energy, such as the steam engine, internal combustion engine, and electricity, constituted a radical form of hardware-augmenting technical change. It is possible that such innovations will also appear in the future. Perhaps a more accurate way to model a hypothetical future hardware revolution, though, rather than embracing unbounded hardware-augmenting technical change, would be to allow for the accumulation of a new, much more energy efficient form of programmable hardware M (Growiec 2019).

Quick algebra proves that this result remains intact also when automation happens only in production or only in R&D, or if there is no R&D capital.

Hardware-augmenting technical change with partial or no automation. In the long-run limit, as all tasks will eventually get mechanized but a fraction of essential production and R&D tasks will forever remain immune to automation, we may approximate $X \approx A^\kappa \alpha K$ and $S \approx \gamma AhN$.

I find that as $A \to \infty$ and $K \to \infty$, asymptotically K grows faster than $A^{1-\kappa}$. It follows that:

$$Y = F(A^\kappa \alpha K, \gamma AhN) = \gamma AhNF\left(\frac{\alpha K}{A^{1-\kappa}\gamma hN}, 1\right) \to \gamma a_N AhN, \tag{39}$$

$$\dot{A} = \Phi(A^\kappa \alpha K, \gamma AhN) = \gamma AhN\Phi\left(\frac{\alpha K}{A^{1-\kappa}\gamma hN}, 1\right) \to \gamma b_N AhN, \tag{40}$$

$$\dot{K} \approx s\gamma a_N AhN - \delta K. \tag{41}$$

The economy converges to a balanced growth path where the long-run GDP growth rate is determined by the pace of technical change:

$$g = g_A = g_K = \gamma b_N hN. \tag{42}$$

Comparing this result to the corresponding case without hardware-augmenting technical change (Eq. (18)), I find that now we do not have a dual growth engine anymore. This is because hardware-augmenting technical change, coupled with the accumulation of R&D capital, resolves the scarcity of hardware, and thus the only remaining scarce factor of production is software (human cognitive work). In such circumstances, the pace of hardware-augmenting technical change, as long as it is positive, is not relevant for the long-run GDP growth rate, which remains constrained by the scarce supply of human cognitive work hN. The fundamental engine of growth is then R&D, generating labor-augmenting technical developments.

I also observe that if there were also positive R&D spillovers ("standing on shoulders" effects) on top of hardware-augmenting technical change ($\phi \in (0, 1)$, then the R&D equation would have been explosive again. To see this, take

$$\dot{A} = A^\phi \Phi(A^\kappa \alpha K, \gamma AhN) = A^{1+\phi}\gamma hN\Phi\left(\frac{\alpha K}{A^{1-\kappa}\gamma hN}, 1\right) \to \gamma b_N A^{1+\phi} hN. \tag{43}$$

This equation implies an ever increasing growth rate of technology, $g_A = A^\phi \gamma b_N hN$, which – given that output Y is proportional to A – implies explosive, super-exponential economic growth.

Hardware-augmenting technical change with partial or no automation and no R&D capital. Let us now check how potent hardware-augmenting technical change is for generating long-run growth under the relatively most adverse circumstance: when there is partial or no automation and no R&D capital. In this scenario, in the long-run limit there will be full mechanization but no automation in production ($X \approx \alpha K, S \approx \gamma AhN$). With $K \to \infty$ and $A \to \infty$ I obtain that K grows faster than $A^{1-\kappa}$ and thus:

$$Y \approx F(A^\kappa \alpha K, \gamma AhN) = \gamma AhNF\left(\frac{\alpha K}{A^{1-\kappa}\gamma hN}, 1\right) \to \gamma a_N AhN, \tag{44}$$

$$\dot{A} \approx \Phi(A^\kappa \zeta N, \gamma AhN) = A^\kappa \zeta N\Phi\left(1, \frac{\gamma hA^{1-\kappa}}{\zeta}\right) \to b_K A^\kappa \zeta N, \tag{45}$$

$$\dot{K} \approx s\gamma a_N AhN - \delta K. \tag{46}$$

In this scenario, due to $\kappa < 1$ technology A grows sub-exponentially (Groth, Koch, and Steger 2010; Jones 1995), and so does capital and output. The pace of hardware-augmenting technical change, while important over the transition, becomes irrelevant for the GDP growth rate in the long-run limit.

If there were also sufficiently strong R&D spillovers ("standing on shoulders" effects) on top of hardware-augmenting technical change, though ($\phi > 1-\kappa$), then the R&D equation would have been explosive again. To see this, take

$$\dot{A} = A^{\phi}\Phi(A^{\kappa}\zeta N, \gamma AhN) = A^{\phi+\kappa}\zeta N\Phi\left(1, A^{1-\kappa}\frac{\gamma h}{\zeta}, 1\right) \to b_K A^{\phi+\kappa}\zeta N. \tag{47}$$

This implies an ever increasing growth rate of technology, $g_A = A^{\phi+\kappa-1}b_K\zeta N$, which – given that output Y is proportional to A – implies super-exponential, explosive economic growth.

In the case of positive but weak R&D spillovers ($\phi \in (0, 1-\kappa)$), the aforementioned sub-exponential growth result remains intact.

In the intermediate knife-edge case $\phi = 1-\kappa$, economic growth becomes exactly exponential in the limit, and the GDP growth rate converges to $g = b_K\zeta N$, determined by the pace of R&D, which is in turn set by the supply of human physical labor in the R&D sector, ζN.

7 Summary and concluding remarks

In the current paper I have provided a theoretical investigation of the prospective sources of long-run economic growth in the future. I have formulated a range of predictions conditional on certain key assumptions regarding automatability of production and R&D tasks and structure of the R&D process. The results follow from observing the dynamics of the relatively scarce factor of production in the long-run limit. When the scarce factor is human cognitive work, which is not accumulable per capita, then the key source of growth is labor-augmenting technological progress, provided by R&D. However, if all essential tasks production or R&D tasks will be subject to automation, then labor (human cognitive work) will eventually give way to capital (programmable hardware) as the scarce factor of production. Then the key source of growth will be the accumulation of programmable hardware and growth will accelerate significantly, perhaps even by an order of magnitude.

Each of these scenarios, plus a few additional ones included as robustness checks, places our global future somewhere along the axis: secular stagnation – continued growth at the current pace – growth acceleration – technological singularity. The results are summarized in Table 1.

The big remaining question is, which of these scenarios is most likely to happen in reality? Let me try to cautiously answer this question by ruling out (or, to phrase it in the Bayesian spirit, by reducing the prior probability of) the scenarios which appear *least* probable given their theoretical underpinnings and historical evidence (Davidson 2021).

First, in my opinion the scenarios with unbounded hardware-augmenting technical change are dubious because of their inconsistency with the laws of thermodynamics. I do admit, however, that *bounded* hardware-augmenting technical change may occur in the event of a new technological revolution in hardware, amounting to the emergence of more efficient ways of performing physical action (and in particular, computation) compared to the ones we know today (e.g., nanotechnology, quantum computing, breakthrough in solar power, fusion power, etc.). For this reason I would not dismiss these scenarios entirely but look at their asymptotic properties with due caution. Second, I also think that both anecdotal and systematic econometric evidence speaks in favor of presence of capital (in particular, programmable hardware) in the R&D process. This narrows down the set of most probable scenarios to the five baseline cases.

To further discriminate among these five scenarios, one needs to assess whether full automation of all essential tasks is technologically feasible (and economically viable). This is a deep question related to the possibility of creating a range of AI algorithms covering all essential domains in which the human brain is used, and indeed possibly unifying their functions in an overarching superhuman artificial general intelligence (Bostrom 2014). Discussions on these issues are ongoing and stretch far beyond the domain of economics. If the answer to this question turns out to be positive (which, as Bostrom documents, is commonly expected by AI researchers), we will be realizing the top-most scenario in Table 1. On the one hand, this

Table 1: Summary of results.

Scenario	Growth engine	Growth rate
	Baseline (with R&D capital)	
Full automation in production and R&D	K acc	$g = s\alpha a_K - \delta$
Full automation in production	K acc	$g = s\alpha a_K - \delta$
Full automation in R&D	K acc	$g = s\alpha a_K - \delta$
Partial or no automation, $\phi = 0$	K acc + LATC	Eq. (18)
Partial or no automation, $\phi \in (0, 1]$	K acc	$g = s\alpha a_K - \delta$
	Without R&D capital	
Full automation in production and R&D	K acc	$g = s\alpha a_K - \delta$
Full automation in production	K acc	$g = s\alpha a_K - \delta$
Full automation in R&D	K acc	$g = s\alpha a_K - \delta$
Partial or no automation, $\phi \in [0, 1)$	LATC*	Secular stagnation
Partial or no automation, $\phi = 1$	LATC**	$g = b_K \zeta N$
	With hardware-augmenting technical change	
Full automation in production and R&D	K acc + KATC	Explosive growth
Full automation in production	K acc + KATC	Explosive growth
Full automation in R&D	K acc + KATC	Explosive growth
. . . With R&D capital. . .		
Partial or no automation, $\phi = 0$	LATC	$g = \gamma b_N h N$
Partial or no automation, $\phi \in (0, 1]$	K acc + LATC	Explosive growth
. . . Without R&D capital. . .		
Partial or no automation, $\phi \in [0, 1 - \kappa)$	LATC	Secular stagnation
Partial or no automation, $\phi = 1 - \kappa$	LATC	$g = b_K \zeta N$
Partial or no automation, $\phi \in (1 - \kappa, 1]$	LATC	Explosive growth

LATC; labor-augmenting technical change, KATC; capital-augmenting technical change, *semi-endogenous R&D-based growth; **fully endogenous R&D-based growth.

scenario expects a (potentially massive, order of magnitude) acceleration in GDP growth.[12] On the other hand, though, this growth acceleration is achieved only as the human contribution to overall production and R&D output falls towards zero in percentage terms, marginalized or outright replaced by the productive contribution of programmable machines and their software. The human labor share of output gradually falls to zero.[13]

If full automation of all essential tasks turns out to be impossible, though, then the most likely scenario will probably be the one in line 4 in Table 1. In this scenario output growth will be always constrained by the pace of labor-augmenting technical change, the only force able to systematically increase the effective supply of the scarce factor of production: human cognitive work performing the non-automatable tasks. The declining supply of human R&D labor will be counteracted by the accumulation of programmable R&D capital and its software, creating a dual growth engine (labor-augmenting technical change plus R&D capital accumulation). GDP growth rates will probably remain in the same ballpark as the ones observed currently, with doubling times of the order of 20–30 years. The human labor share of output will eventually stabilize at a positive value.

12 The hypothesis that machine intelligence substituting human cognitive work may lead to an order-of-magnitude acceleration in GDP growth was first formulated by Hanson (2001).

13 If furthermore the global R&D sector, run either by people or the superhuman AI, finds a way to ease the growing scarcity of programmable hardware by the means of hardware-augmenting technical change, the world would be heading towards technological singularity.

In sum, this study has drawn the span of potential long-run growth outcomes for the digital era in which production and R&D processes can be potentially automated, pointing to a likely growth acceleration as more and more processes become fully automated in the future. In further research, the current paper could be expanded into a quantitative, numerical assessment of relative importance of the considered mechanisms over the coming decades. However, this must necessarily involve a fair amount of speculation. How long is the long run? How long is the transition period going to be? At which point will we realize that human cognitive work and pre-programmed software, previously complementary because many tasks required the human input, have already become broadly substitutable? When will – if at all – the accumulation of programmable hardware overtake labor-augmenting technical change as the key engine of growth? As Niels Bohr used to say, "it is difficult to predict, especially the future".

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